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SOLVING PARALLEL MOVING METHOD USING PENTAGON FUZZY NUMBERS

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ABSTRACT

A parallel moving method is proposed to find an optimal solution to the fuzzy problem using pentagon fuzzy number. In the fuzzy assignment problem, we discussed the membership functions of the fuzzy cost matrix both in the monotonically increasing and decreasing function.

KEYWORDS: Assignment Problem, Pentagon Fuzzy Number, Parallel Moving Method

MATHEMATICS SUBJECT CLASSIFICATION CODE (2010): 03E72, 90B80

INTRODUCTION

Assignment problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in industry and other applications. In an assignment**, n** jobs are to be performed by n persons of depending on their efficiency to do the job. In this problem c_{ij} denotes the cost assigning the j^{th} job to the j^{th} person. The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate. The AP introduced by Votaw and Orden [16] can be solved using the linear programming technique. However, due to its highly degeneracy nature a specially designed algorithm, widely known as Hungarian method proposed by Kuhn [9] cost in many real life applications are not deterministic numbers. The fuzzy assignment problem (FAP) is more realistic than the AP because most real environments are uncertain. In this paper, we investigate more realistic assignment problem with fuzzy cost $\tilde{c}_{ij}$ since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also a fuzzy number. Chen [6] proved some theorems and proposed a fuzzy assignment model that considers all the individuals to have same skills. Lin and Wen [10] investigated a fuzzy assignment problem in which the cost depends on the quality of the job. An integer fuzzy transportation problem was solved in Tada and Ishii [15]. Feng and yang [8] studied a two objective fuzzy K-cardinality AP. Long-Sheng Huang and Guang-Hui-xu [11] proposed solution procedure for the AP with restriction of qualification. By the max-min criterion suggested by Belman and Zadeh [3], the fuzzy assignment problem can be treated as a mixed integer non-linear programming problem. Shigeno et.al. [14] proposed a solution procedure for the fractional assignment problem. We proposed parallel moving method for finding an optimal solution to the FAP considered in Lin and Wen [10]. In this paper, section 2 deals with preliminaries. In section 3, we proposed new ranking for the pentagon fuzzy number which is based on the incenter of the centroids. In section 4, we discussed a brief note on parallel moving method. In section 5, the effectiveness of the proposed method is illustrated by means of an example.

2. PRELIMINARIES

2.1 Definition: Fuzzy Set

A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse x to the unit interval [0,1].A fuzzy set $\tilde{A}$ is set of ordered pairs $\{x, \mu_A(x) / x \in R\}$ where $\mu_A(x): R \to [0,1]$ is upper semi- continuous function $\mu_A(x)$ is called membership function of the fuzzy set.

2.2 Definition: Fuzzy Number

A fuzzy number f in the real line R is a fuzzy set $f: R \to [0,1]$ that satisfies the following properties.

(i) f Is piecewise continuous.

(ii) There exists an $x \in$ R such that $f(x) = 1$.

(iii) f is convex,(i.e).,if $x_1, x_2 \in R$ and $a \in [0,1]$ then $f(\lambda x_1 + (1-\lambda) x_2) \geq f(x_1) \wedge f(x_2)$.

2.3 Pentagon Fuzzy Number

A pentagon fuzzy number $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership is given below.

$$\mu_{\tilde{A}_P}(x) = \begin{cases} 0 & , \quad x < a_1 \\ \frac{1}{2}\left[\frac{x-a_1}{a_2-a_1}\right] & , \quad a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\left[\frac{y-a_2}{a_3-a_2}\right] & , \quad a_2 \leq x \leq a_3 \\ 1 - \frac{1}{2}\left[\frac{a_4-y}{a_4-a_3}\right] & , \quad a_3 \leq x \leq a_4 \\ \frac{1}{2}\left[\frac{a_5-x}{a_5-a_4}\right] & , \quad a_4 \leq x \leq a_5 \\ 0 & , \quad x > a_5 \end{cases}$$

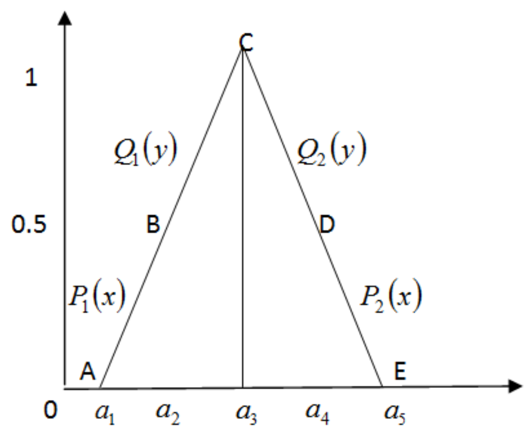

Figure 1: Graphical Representation of a Normal Pentagon Fuzzy Number for $x \in$ [0, 1]

Definition: Canonical Pentagon Fuzzy Number

$\tilde{A}_P$ Is called a Canonical pentagon fuzzy number if it is a closed and bounded pentagon fuzzy number and its membership function is strictly increasing on the interval [a_2, a_3] and strictly decreasing on the interval [a_3, a_4].

Remark

Pentagon fuzzy number $\tilde{A}_P$ is the ordered quadruple ($P_1(x), Q_1(y), Q_2(y), P_2(x)$) for $x \in [0,0.5]$ and y∈ [0.5,1] where,

$$P_1(x) = \frac{1}{2}\left[\frac{x-a_1}{a_2-a_1}\right] \quad , \quad P_2(x) = \frac{1}{2}\left[\frac{a_5-x}{a_5-a_4}\right]$$

$$Q_1(y) = \frac{1}{2}+\frac{1}{2}\left[\frac{y-a_2}{a_3-a_2}\right] \; , \; Q_2(y) = 1-\frac{1}{2}\left[\frac{a_4-y}{a_4-a_3}\right]$$

2.5 Definition

A positive pentagon fuzzy number $\tilde{A}_P$ is denoted as $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where $a_i's > 0$ for all i=1,2,3,4,5 (e.g.): $\tilde{A}_P$ = (3,5,7,9,11).

2.6 Definition

A negative pentagon fuzzy number $\tilde{A}_P$ is denoted as $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where $a_i's < 0$ for all i=1,2,3,4,5 (e.g): $\tilde{A}_P$ = (-5,-4,-3,-2,-1).

2.7 Definition

Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be two pentagon fuzzy number if $\tilde{A}_P$ is identically equal to $\tilde{B}_P$ only if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$, $a_4 = b_4$, $a_5 = b_5$.

3. RANKING OF PENTAGON FUZZY NUMBERS

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function R: F(R)→R where F(R) is a set of fuzzy numbers defined on set of real numbers which maps each fuzzy number into a real numbe

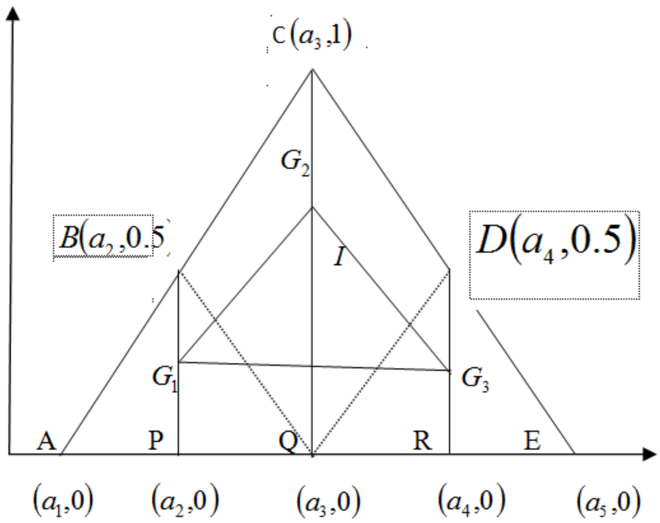

Figure 2: Normalized Pentagon Fuzzy Number

The centroid of a pentagon fuzzy number is considered to be the balancing point of the pentagon (figure 2). Divide the pentagon into three plane figures. In these three plane first figure is a triangular ABQ, second figure is a square QBCD and third is again a triangle QDE. Let the centroid of the three plane figures be G_1, G_2, G_3 respectively. The Incentre of the centroid $\gamma\gamma G_1$, G_2, G_3 is taken as the point of reference to define the ranking of normalized pentagon fuzzy numbers. Consider a normalized pentagon fuzzy number

$\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ the centroid of the three plane figures are

$G_1 = \{[a_1 + a_2 + a_3]/3, 1/6\}$

$G_2 = \{[a_2 + 2a_3 + a_4]/4, 1/2\}$

$G_3 = \{[a_3 + a_4 + a_5]/3, 1/6]\}$ respectively.

$$I_{\tilde{A}_P} = \left\{ \frac{\alpha_{\tilde{A}_P}\left[\frac{a_1+a_2+a_3}{3}\right] + \beta_{\tilde{A}_P}\left[\frac{a_2+2a_3+a_4}{4}\right] + \gamma_{\tilde{A}_P}\left[\frac{a_3+a_4+a_5}{3}\right]}{\alpha_{\tilde{A}_P}+\beta_{\tilde{A}_P}+\gamma_{\tilde{A}_P}}, \frac{\alpha_{\tilde{A}_P}\left[\frac{1}{6}\right] + \beta_{\tilde{A}_P}\left[\frac{1}{2}\right] + \gamma_{\tilde{A}_P}\left[\frac{1}{6}\right]}{\alpha_{\tilde{A}_P}+\beta_{\tilde{A}_P}+\gamma_{\tilde{A}_P}} \right\}$$

Where,

$$\alpha_{\tilde{A}_p} = \frac{\sqrt{(a_4+4a_5-2a_3-3a_2)^2+8}}{12}, \quad \beta_{\tilde{A}_p} = \frac{\sqrt{(a_1+a_2-a_4-a_5)^2}}{3} \quad \text{and}$$

$$\gamma_{\tilde{A}_p} = \frac{\sqrt{(4a_1+a_2-2a_3-3a_4)^2+8}}{12}$$

Now,

$$R_{\tilde{A}_p} = \sqrt{\bar{x}_0^{\,2} + \bar{y}_0^{\,2}}$$

4. PARALLEL MOVING METHOD

A company producing 3 different products having 3 workers, who are capable of producing any of the 3 products, works effectively. The cost required for each worker for producing each of the products are taken in the form of matrix. In this case, the cost are no longer deterministic numbers and we will denote them as $\tilde{c}_{ij}$. Further, we define as α_{ij} the least cost associated with the worker i performing the job j and as $\beta_{ij,}$ the least cost associated with worker i performing the job j at the highest quality. Without loss of generality, we may assume that $\beta_{ij}>\alpha_{ij}>0$. Further we define the quality matrix $[q_{ij}]$, where q_{ij} represents the highest quality associated with the worker i performing the job j. Any expense exceeding β_{ij} is useless. Since the quality no longer be increased as its upper limit q_{ij}. An increase in cost does not increase the quality. Let $\tilde{C}_T$ denote the total cost, which related to the job performance of the manager, and numbers 'a' and 'b' are defined as the lower and upper bounds of total cost, respectively. We define the membership function of $\tilde{C}_T$ as the linearly monotonic increasing function and linearly monotonic decreasing function and we use the notation <a,b> to denote the fuzzy interval $\tilde{C}_T$. Numbers 'a' and 'b' are constants and are subjectively chosen by the manager.By the idea of Werners a number less than or equal to the minimum assignment of the matrix $[\alpha_{ij}]$ as 'a' and a number larger than or equal to the maximum assignment of the matrix $[\beta_{ij}]$ as 'b' are taken. The membership function of $\tilde{C}_T$ for 4 different categories is given below.(i,e) (a_1,a_2),(a_2,a_3),(a_3,a_4),(a_4,a_5).

The Membership Function for $\tilde{C}_T$ in the Interval (a_1, a_2) is

$$
\begin{array}{l}
\qquad\qquad\qquad 0.5 \quad ; \quad \text{if} \quad C_T \le a \\
\mu(C_T) = \mu\sum_{i=1}^{n}\sum_{j-1}^{n} C_{ij}X_{ij} = \dfrac{b-C_T}{\beta_{ij}-\alpha_{ij}}; \quad \text{if} \quad a \le C_T \le b \\
\qquad\qquad\qquad 0 \quad ; \quad \text{if} \quad C_T \ge b
\end{array}
$$

The membership function for $\tilde{C}_T$ in the interval (a_2,a_3) is

$$
\begin{array}{l}
\qquad\qquad 1 \quad ; \quad \text{if} \quad C_T \le a \\
\mu\sum_{i=1}^{n}\sum_{j-1}^{n} C_{ij}X_{ij} = \dfrac{b-C_T}{\beta_{ij}-\alpha_{ij}}; \quad \text{if} \quad a \le C_T \le b \\
\qquad\qquad 0.5 \quad ; \quad \text{if} \quad C_T \ge b
\end{array}
$$

The membership function for $\tilde{C}_T$ in the interval (a_3,a_4) is

$$\mu\sum_{i=1}^{n}\sum_{j-1}^{n}C_{ij}X_{ij} = \begin{cases} 1 & ; \text{ if } \quad C_T \le a \\ \dfrac{b-C_T}{\beta_{ij}-\alpha_{ij}}; & \text{ if } \quad a \le C_T \le b \\ 0.5 & ; \text{ if } \quad C_T \ge b \end{cases}$$

The membership function for $\tilde{C}_T$ in t he interval (a_4,a_5) is

$$\mu(C_T) = \mu\sum_{i=1}^{n}\sum_{j-1}^{n}C_{ij}X_{ij} = \begin{cases} 0.5 & ; \text{ if } \quad C_T \le a \\ \dfrac{b-C_T}{\beta_{ij}-\alpha_{ij}}; & \text{ if } \quad a \le C_T \le b \\ 0 & ; \text{ if } \quad C_T \ge b \end{cases}$$

Further, we define

Maximize f = $\dfrac{b-\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}x_{ij}}{b-a+\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}}$

subject to $\sum_{j=1}^{n}x_{ij} = 1$; i= 1,2, 3..........n

$\sum_{i=1}^{n}x_{ij} = 1$; j= 1,2,3.. .....n

$x_{ij} = \{0,1\}$; i= 1,2,..........n , j= 1,2,3..........n

Where, $\gamma_{ij} = \dfrac{\beta_{ij}-\alpha_{ij}}{q_{ij}}$

5. NUMERICAL EXAMPLE

In the consultant company, the manager restricts the total cost to a range as his fuzzy goal. We assume a minimum cost for a worker to perform a job, and greater cost spent may result in higher quality until it reaches an upper bound.

$$\tilde{C}_{ij} = \begin{bmatrix} (7,9,11,13,15) & (13,15,17,19,21) & (6,7,8,9,10) \\ (5,7,9,11,13) & (3,5,7,9,11) & (8,10,12,14,16) \\ (11,12,13,14,15) & (14,15,16,17,18) & (13,14,15,16,17) \end{bmatrix}$$

The main objective of the FAP is to find an assignment to the given FAP which minimize the total cost of the workers and maximize the job performance of the manager. The costs in the AP's are uncertain.This leads to the use of pentagon fuzzy number for representing these imprecise values.

Step 1: Construct the APs (L) and (U) from the given FAP. Then, solve them by the Hungarian method. Say X_0 is an optimal solution to the problem (L) and Y_0 is an optimal solution to the problem (U).

Step 2: If $X_0 = Y_0$, then X_0 is an optimal solution to the given FAP, Go to step 7. If not, go to step 3.

Step 3: Fix U_0 where $U_0 = X_0$ or Y_0 as an initial solution to the given FAP. Then, compute the value of $f(U_0)$, f_0.

Step 4: Compute the dual variables (MODI indices) u_i and v_j for all i and j using the relation $u_i+v_j+\alpha_{ij}+f_0\gamma_{ij} = 0$, for allotted cells by taking $u_i=0$, for all i.

Step 5: Construct MODI index table for the initial solution, U_0 to the FSP. Then, compute $\delta_{ij} = u_i+v_j+\alpha_{ij}+f_0\gamma_{ij}$ for all non- allotted cells. If $\delta \geq 0$, for all non-alloted cells, go to the Step 7. If not, go to the Step 6.

Step 6: Find a cell having the most negative value of δ_{ij}. Say (p,s). Then,

(a) If the assignments of p and s in the initial solution U_0 are (p,r) and (q,s) and if

$(\gamma_{pr} + \gamma_{qs}) - (\gamma_{ps} + \gamma_{qr}) \leq 0$ and $(\alpha_{pr}+\alpha_{qs}) - (\alpha_{ps} + \alpha_{qr}) \geq 0$, assign (p,s) and (q,r) and obtain an improve solution U1 = (U0 –{(p,r),(q,s)}) $\cup$ {(p,s),(q,r)}. Then go to step 7 for next iteration.

If the one of the conditions "$(\gamma_{pr} + \gamma_{qs}) - (\gamma_{ps} + \gamma_{qr}) \leq 0$ and $(\alpha_{pr}+\alpha_{qs}) - (\alpha_{ps} + \alpha_{qr}) \geq 0$"is not satisfied, go to step 6(b).

(b) Find a cell in the qth row, say (q,t) which satisfies the conditions

$(\gamma_{pr} + \gamma_{qs} + \gamma_{zt}) - (\gamma_{ps} + \gamma_{qt} + \gamma_{zr}) \leq 0$ and $(\alpha_{pr} + \alpha_{qs} + \alpha_{zt}) - (\alpha_{ps} + \alpha_{qt} + \alpha_{zr}) \geq 0$ where (z,t) is the assignment of t in the initial solution U_0,assign (p,s) , (q,t) and (z,r) and obtain an improve solution. $U_1 = (U_0$ – {(p,r),(q,s),(z,t)}) $\cup${(p,s),(q,t),(z,r)}.Then go to step 2 for next iteration. If the above conditions are not satisfied then go to step 6(c).

(c) Continue for finding an assignment to the rth column as like as in the step 6(b).After finite number of steps, an improve solution is obtained. Then, go to the step 2. for the next iteration.

Step 7: The current solution is an optimal solution to the given FAP. We stop the computation process. The current value of f is the maximum value of f.

Considering the interval (a_1, a2)

$$\tilde{c}_{ij} = \begin{bmatrix} (7,9) & (13,15) & (6,7) \\ (5,7) & (3,5) & (8,10) \\ (11,12) & (14,15) & (13,14) \end{bmatrix}$$

The quality matrix $q_{ij} = \begin{bmatrix} 0.4 & 0.1 & 0.3 \\ 0.4 & 0.4 & 0.4 \\ 0.3 & 0.3 & 0.1 \end{bmatrix}$

$\alpha_{ij} = \begin{bmatrix} 7 & 13 & 6 \\ 5 & 3 & 8 \\ 11 & 14 & 13 \end{bmatrix}$; $\beta_{ij} = \begin{bmatrix} 9 & 15 & 7 \\ 7 & 5 & 10 \\ 12 & 15 & 14 \end{bmatrix}$ and

$$\gamma_{ij} = \begin{bmatrix} 5 & 20 & 3.3 \\ 5 & 5 & 5 \\ 3.3 & 3.3 & 10 \end{bmatrix}$$

Solve them by Hungarian method, the optimal solution to (L) is (1,3) (2,2) (3,1) and optimal solution to (U) is (1,3) (2,2) (3,1) and (1,3) (2,1) (3,2)

Case (i): The initial solution: (1, 3) (2, 2) (3, 1)

Now the value of 'f ' for the above allotment is 0.6 then, by the step 4 the assignment is common to (L) and (U) then optimal value of f is 0.6.

Case (ii): The initial solution: (1, 3) (2, 1) (3, 2)

The value of f = 0.5

Take, $u_i = 0$ for all i now, $v_j = -\alpha_{ij} - f\gamma_{ij}$; for all allotted cells.

Modi-Index Table: (to check the optimality)

Table 1

	v_1 = -7.5	v_2 = -15.65	V_3= -7.65
u_1=0	2 5	7.35 20	X 3.3
u_2=0	X 5	-10.15 5	2.85 5
u_3 =0	5.15 3.3	X 3.3	10.35 10

The most negative of δ_{ij} for non-allotted cells is (2,2), since we have the assignment (2,1) (3,2). $(\gamma_{21} + \gamma_{32}) - (\gamma_{22} + \gamma_{31}) = 0 \leq 0$ and $(\alpha_{21}+\alpha_{32}) - (\alpha_{22} + \alpha_{31}) = 5 \geq 0$.

FIRST ITERATION

U_1 = (1, 3) (2, 2) (3, 1). Now, the solution to the fuzzy AP in the interval (a_1,a_2) is (1,3)(2,2)(3,1) and the value of f is 0.6.

By solving similarly for the other three intervals (a_2,a_3),(a_3,a_4),(a_4,a_5). We get the optimal assignment is (1,3)(2,2)(3,1) and value of f is 0.8,0.8,0.6 respectively.

The optimal assignment scheduled is (1, 3) (2, 2) (3, 1) in all the intervals. The maximum job performance in the intervals (a_1, a2), (a_4, a_5) is 0.6. And the max job performance in the intervals (a_2, a_3), (a_3, a_4) is 0.8.

CONCLUSIONS

In this paper, we propose a method namely parallel moving method for solving FAP using pentagon fuzzy number. From the numerical example, we can easily identify the maximum job performance in the particular interval.

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